1. Rabbits were introduced onto an island. The number of rabbits, P, t years after they were introduced is modelled by the equation

$$P = 80e^{\frac{1}{5}t} \qquad t \in \mathbb{R}, t \geq 0$$

(a) Write down the number of rabbits that were introduced to the island.

(1)

(b) Find the number of years it would take for the number of rabbits to first exceed 1000.

(2)

(c) Find $\frac{\mathrm{d}p}{\mathrm{d}t}$.

(2)

(d) Find P when $\frac{\mathrm{d}p}{\mathrm{d}t} = 50$.

(3)
(Total 8 marks)

2. The function f is defined by

$$\mathrm{f}(x) = 1 - \frac{2}{(x+4)} + \frac{x-8}{(x-2)(x+4)}, \quad x \in \mathbb{R}, x \neq -4, x \neq 2$$

(a) Show that $\mathrm{f}(x) = \frac{x-3}{x-2}$

(5)

The function g is defined by

$$g(x) = \frac{e^x - 3}{e^x - 2}, \qquad x \in \mathbb{R}, x \neq \ln 2$$

(b) Differentiate g(x) to show that $g'(x) = \dfrac{e^x}{(e^x - 2)^2}$,

(3)

(c) Find the exact values of x for which g′(x) = 1

(4)
(Total 12 marks)

3. The point P lies on the curve with equation

$$y = 4e^{2x+1}.$$

The y -coordinate of P is 8.

(a) Find, in terms of ln 2, the x -coordinate of P.

(2)

(b) Find the equation of the tangent to the curve at the point P in the form $y = ax + b$, where a and b are exact constants to be found.

(4)
(Total 6 marks)

4. The amount of a certain type of drug in the bloodstream t hours after it has been taken is given by the formula

$$x = De^{-\frac{1}{8}t},$$

where x is the amount of the drug in the bloodstream in milligrams and D is the dose given in milligrams.

A dose of 10 mg of the drug is given.

(a) Find the amount of the drug in the bloodstream 5 hours after the dose is given.
Give your answer in mg to 3 decimal places.

(2)

A second dose of 10 mg is given after 5 hours.

(b) Show that the amount of the drug in the bloodstream 1 hour after the second dose is 13.549 mg to 3 decimal places.

(2)

No more doses of the drug are given. At time T hours after the second dose is given, the amount of the drug in the bloodstream is 3 mg.

(c) Find the value of T.

(3)
(Total 7 marks)

5. A heated metal ball is dropped into a liquid. As the ball cools, its temperature, T °C, t minutes after it enters the liquid, is given by

$$T = 400\,\mathrm{e}^{-0.05t} + 25, \; t \geq 0.$$

(a) Find the temperature of the ball as it enters the liquid.

(1)

(b) Find the value of t for which $T = 300$, giving your answer to 3 significant figures.

(4)

(c) Find the rate at which the temperature of the ball is decreasing at the instant when $t = 50$.
Give your answer in °C per minute to 3 significant figures.

(3)

(d) From the equation for temperature T in terms of t, given above, explain why the temperature of the ball can never fall to 20 °C.

(1)
(Total 9 marks)

6. A particular species of orchid is being studied. The population p at time t years after the study started is assumed to be

$$p = \frac{2800a\mathrm{e}^{0.2t}}{1 + a\mathrm{e}^{0.2t}}, \text{ where } a \text{ is a constant.}$$

Given that there were 300 orchids when the study started,

(a) show that $a = 0.12$,

(3)

(b) use the equation with $a = 0.12$ to predict the number of years before the population of orchids reaches 1850.

(4)

(c) Show that $p = \dfrac{336}{0.12 + \mathrm{e}^{-0.2t}}$.

(1)

(d) Hence show that the population cannot exceed 2800.

(2)
(Total 10 marks)

7. Find, giving your answer to 3 significant figures where appropriate, the value of x for which

(a) $3^x = 5$,

(3)

(b) $\log_2 (2x + 1) - \log_2 x = 2$,

(4)

(c) $\ln \sin x = -\ln \sec x$, in the interval $0 < x < 90°$.

(3)
(Total 10 marks)

8. Every £1 of money invested in a savings scheme continuously gains interest at a rate of 4% per year. Hence, after x years, the total value of an initial £1 investment is £y, where

$$y = 1.04^x.$$

(a) Sketch the graph of $y = 1.04^x$, $x \geq 0$.

(2)

(b) Calculate, to the nearest £, the total value of an initial £800 investment after 10 years.

(2)

(c) Use logarithms to find the number of years it takes to double the total value of any initial investment.

(3)
(Total 7 marks)

9. Find the exact solutions of

(i) $e^{2x + 3} = 6$,

(3)

(ii) $\ln (3x + 2) = 4$.

(3)
(Total 6 marks)

1. (a) $P = 80\mathrm{e}^{\frac{t}{5}}$

$t = 0 \;\Rightarrow\; P = 80\,\mathrm{e}^{\frac{o}{5}} = 80(1) = \underline{80}$ — $\underline{80}$ B1 1

(b) $\mathrm{P} = 1000 \;\Rightarrow\; 1000 = 80\mathrm{e}^{\frac{t}{5}} \;\Rightarrow\; \frac{1000}{80} = \mathrm{e}^{\frac{t}{5}}$ Substitutes $P = 1000$ and rearranges equation to make $\mathrm{e}^{\frac{t}{5}}$ the subject.

$\therefore\; t = 5\ln\left(\frac{1000}{80}\right)$

$t = 12.6286...$ — awrt 12.6 or 13 years A1 2

Note $t = 12$
or $t =$ awrt $12.6 \;\Rightarrow\; t = 12$
will score A0

(c) $\frac{\mathrm{d}P}{\mathrm{d}t} = 16\mathrm{e}^{\frac{t}{5}}$ — $k\mathrm{e}^{\frac{t}{5}}$ and $k \neq 80$.

$16\,\mathrm{e}^{\frac{1}{5}t}$ A1 2

(d) $50 = 16\mathrm{e}^{\frac{t}{5}}$

$\therefore\; t = 5\ln\left(\frac{50}{16}\right)$ $\{5.69717...\}$ — Using $50 = \frac{\mathrm{d}P}{\mathrm{d}t}$ and an attempt to solve to find the value of t or $\frac{t}{5}$

$P = 80\,\mathrm{e}^{\frac{1}{5}\left(5\ln\left(\frac{50}{16}\right)\right)}$ or $P = 80\mathrm{e}^{\frac{1}{5}(5.69717...)}$ — Substitutes their value of t back into the equation for P. dM1

$P = \frac{80(50)}{16} = \underline{250}$ — $\underline{250}$ or awrt 250 A1 3

[8]

2. (a) $f(x) = 1 - \dfrac{2}{(x+4)} + \dfrac{x-8}{(x-2)(x+4)}$

$x \in \mathbb{R}, x \neq -4, x \neq 2.$

$f(x) = \dfrac{(x-2)(x+4) - 2(x-2) + x - 8}{(x-2)(x+4)}$ An attempt to combine to one fraction

Correct result of combining all three fractions A1

$= \dfrac{x^2 + 2x - 8 - 2x + 4 + x - 8}{(x-2)(x+4)}$

$= \dfrac{x^2 + x - 12}{[(x+4)(x-2)]}$ Simplifies to give the correct numerator. Ignore omission of denominator A1

$= \dfrac{(x+4)(x-3)}{[(x+4)(x-2)]}$ An attempt to factorise the numerator. dM1

$= \dfrac{(x-3)}{(x-2)}$ Correct result A1 cso **AG** 5

(b) $g(x) = \dfrac{e^x - 3}{e^x - 2}$ $x \in \mathbb{R}, x \neq \ln 2.$

Apply quotient rule: $\left\{\begin{array}{ll} u = e^x - 3 & v = e^x - 2 \\ \dfrac{du}{dx} = e^x & \dfrac{dv}{dx} = e^x \end{array}\right\}$

$g'(x) = \dfrac{e^x(e^x - 2) - e^x(e^x - 3)}{(e^x - 2)^2}$ Applying $\dfrac{vu' - uv'}{v^2}$

Correct differentiation A1

$= \dfrac{e^{2x} - 2e^x - e^{2x} + 3e^x}{(e^x - 2)^2}$

$= \dfrac{e^x}{(e^x - 2)^2}$ Correct result A1 **AG** cso 3

(c)	$g'(x) = 1 \Rightarrow = \dfrac{e^x}{(e^x - 2)^2} = 1$				
	$e^x = (e^x - 2)^2$	Puts their differentiated numerator equal to their denominator.			
	$e^x = e^{2x} - 2e^x - 2e^x + 4$				
	$\underline{e^{2x} - 5e^x + 4} = 0$	$\underline{e^{2x} - 5e^x + 4}$	A1		
	$(e^x - 4)(e^x - 1) = 0$	Attempt to factorise or solve quadratic in e^x			
	$e^x = 4$ or $e^x = 1$				
	$x = \ln 4$ or $x = 0$	both $x = 0, \ln 4$	A1	4	
					[12]

3.	(a)	$e^{2x+1} = 2$ $2x + 1 = \ln 2$				
		$x = \dfrac{1}{2}(\ln 2 - 1)$		A1	2	
	(b)	$\dfrac{dy}{dx} = 8e^{2x+1}$		B1		
		$x = \dfrac{1}{2}(\ln 2 - 1) \Rightarrow \dfrac{dy}{dx} = 16$		B1		
		$y - 8 = 16\left(x - \dfrac{1}{2}(\ln 2 - 1)\right)$				
		$y = 16x + 16 - 8 \ln 2$		A1	4	
						[6]

4.	(a)	$D = 10, t = 5, x = 10e^{-\frac{1}{8}\times 5}$ $= 5.353$	awrt	A1	2
	(b)	$D = 10 + 10e^{-\frac{5}{8}}, t = 1, x = 15.3526... \times e^{-\frac{1}{8}}$ $x = 13.549$ (*)		A1cso	2

Alt.	(b)	$x = 10e^{-\frac{1}{8}\times 6} + 10e^{-\frac{1}{8}\times 1}$ $\quad x = 13.549$ (*)	A1cso	
		(Main scheme is for $(10 + 10e^{-\frac{5}{8}})e^{-\frac{1}{8}}$, or {10 + their(a)} $e^{-\frac{1}{8}}$		
		N.B. The answer is given. There are many correct answers seen which deserve M0A0 or M1A0		
	(c)	$15.3526...\,e^{-\frac{1}{8}T} = 3$		
		$e^{-\frac{1}{8}T} = \dfrac{3}{15.3526...} = 0.1954...$		
		$-\dfrac{1}{8}T = \ln 0.1954...$		
		T = 13.06... or 13.1 or 13	A1	3
		1st M is for $(10+10e^{-\frac{5}{8}})e^{-\frac{T}{8}} = 3$ o.e.		
		2nd M is for converting $e^{-\frac{T}{8}} = k$ $(k > 0)$ to $-\dfrac{T}{8} = \ln k$. This is independent of 1st M.		
		Trial and improvement: as scheme, correct process for their equation (two equal to 3 s.f.) A1 as scheme		

[7]

5.	(a)	425 °C	B1	1
	(b)	$300 = 400e^{-0.05\,t} + 25 \quad \Rightarrow 400e^{-0.05\,t} = 275$		
		sub. T = 300 and attempt to rearrange to $e^{-0.05t} = a$, where $a \in Q$		
		$e^{-0.05t} = \dfrac{275}{400}$	A1	
		correct application of logs		
		t = 7.49	A1	4
	(c)	$\dfrac{dT}{dt} = -20\,e^{-0.05t}$ $\quad$ (for $ke^{-0.05t}$)	A1	
		At t = 50, rate of decrease = (±) 1.64 °C / min	A1	3

(d) $T > 25$, (since $e^{-0.05t} \to 0$ as $t \to \infty$) B1 1

[9]

6. (a) Setting $p - 300$ at $t = 0 \Rightarrow 300 = \dfrac{2800a}{1+a}$

(300 = 2500a); a = 0.12 (c.s.o.) (*) dM1 A1 3

(b) $1850 = \dfrac{2800(0.12)e^{0.2t}}{1+0.12e^{0.2t}}$; $e^{0.2t} = 16.2\ldots$ M1A1

Correctly taking logs to 0.2 t = ln k

t = 14 (13.9..) A1 4

(c) Correct derivation: B1 1

(Showing division of num. and den. by $e^{0.2t}$; using a)

(d) Using $t \to \infty$, $e^{-0.2t} \to 0$,

$p \to \dfrac{336}{0.12} = 2800$ A1 2

[10]

7. (a) $\log 3^x = \log 5$

taking logs

$x = \dfrac{\log 5}{\log 3}$ or $x \log 3 = \log 5$ A1

= 1.46 cao A1 3

(b) $2 = \log_2 \dfrac{2x+1}{x}$

$\dfrac{2x+1}{x} = 4$ or equivalent; B1

4

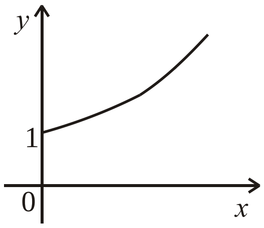
$2x + 1 = 4x$

multiplying by x to get a linear equation

$x = \dfrac{1}{2}$ A1 4

	(c)	sec x = 1/ cos x	B1	
		sin x = cos x $\Rightarrow$ tan x = 1 x = 45	A1	3
		use of tan x		

[10]

8.	(a)			
		Shape	B1	
		domain, intercept	B1	2
	(b)	£800 × 1.04^{10} ≈ £1184	A1	2
		cao		
	(c)	$1.04^x = 2$		
		$x = \dfrac{\ln 2}{\ln 1.04} \approx 18$ (years)	A1	3
		accept 17.7, 17 years 8 months		

[7]

9.	(i)	$e^{2x+3} = 6$		
		$2x + 3 = \ln 6$		
		$x = \dfrac{1}{2}(\ln 6 - 3)$	A1	3
	(ii)	$\ln(3x + 2) = 4$		
		$3x + 2 = e^4$		
		$x = \dfrac{1}{3}(e^4 - 2)$	A1	3

[6]

1. This question was well answered by the overwhelming majority of candidates who demonstrated their confidence in working with exponentials.

Part (a) was almost universally answered correctly, although a few candidates did try to substitute t = 1 into the equation for P in order to find the number of rabbits introduced to the island.

In part (b), most candidates were able to use natural logarithms in order to find t = 12.6 or t = 12.63. Although the expected answer was 13 years, any answer that rounded to 12.6 years was also accepted. Those candidates who continued to round their answer down to 12 or stated “in the 12th year” were not awarded the final accuracy mark as the question required candidates to find the number of years for the number of rabbits to exceed 1000. A few candidates applied a trial and error method in this part and were usually successful in gaining both marks.

In part (c), most candidates correctly stated $\frac{\mathrm{d}P}{\mathrm{dt}}$ as $16\mathrm{e}^{\frac{1}{5}t}$. Common errors in this part were candidates giving answers of the form $k\,t\,\mathrm{e}^{\frac{1}{5}t}$ or $16\,\mathrm{e}^{-\frac{4}{5}t}$. A few candidates tried to apply the product rule for differentiation and usually struggled to gain both marks.

In part (d), the vast majority of candidates equated their $\frac{\mathrm{d}P}{\mathrm{d}t}$ found in part (c) to 50 and proceeded to solve for t. A number of candidates failed at this point to use their value for t to find P as required in the question. It was pleasing to see a significant minority of candidates who deduced that $P = 80\mathrm{e}^{\frac{1}{5}t} = 5 \times 16\mathrm{e}^{\frac{1}{5}t} = 5 \times \frac{\mathrm{d}P}{\mathrm{d}t} = 250$.

2. Many candidates were able to obtain the correct answer in part (a) with a significant number of candidates making more than one attempt to arrive at the answer given in the question. Those candidates who attempted to combine all three terms at once or those who combined the first two terms and then combined the result with the third term were more successful in this part. Other candidates who started by trying to combine the second and third terms had problems dealing with the negative sign in front $\frac{2}{x+4}$ and usually added $\frac{2}{(x+4)}$ to $\frac{x-8}{(x-2)(x+4)}$ before combining the result with 1. It was pleasing to see that very few candidates used $(x + 4)^2(x - 2)$as their common denominator when combining all three terms.

In part (b), most candidates were able to apply the quotient rule correctly but a number of candidates failed to use brackets properly in the numerator and then found some difficultly in arriving at the given answer.

In part (c), many candidates were able to equate the numerator to the denominator of the given fraction and many of these candidates went onto form a quadratic in e^x which they usually solved. A significant number of candidates either failed to spot the quadratic or expanded $(\mathrm{e}^x - 2)^2$ and then took the natural logarithm of each term on both sides of their resulting equation.

In either or both of parts (b) and (c), some candidates wrote e^{x^2} in their working instead of e^{2x} Such candidates usually lost the final accuracy mark in part (b) and the first accuracy mark in part (c).

3. Part (a) was usually completed successfully and the great majority were able to take logs correctly to find x. In part (b), most could differentiate correctly and evaluate $\frac{dy}{dx}$ to find the gradient of the tangent. A few failed to evaluate $\frac{dy}{dx}$, giving a non-linear equation for the tangent, and this lost the last three marks. The majority demonstrated a correct method. However the final mark was often lost. Incorrect removal of the brackets, leading to $y = 16x - 8 \ln 2$, was frequently seen and if, as here, the question asks for exact values of a and b, giving $b = 10.45$ loses the final mark, unless the exact solution, $18 - 8 \ln 2$, is also given.

4. Part (a) was well answered, although candidates who gave the answer to 3 significant figures lost a mark.

In part (b) those candidates who realised that $x = 15.3526..e^{-\frac{1}{8}}$ usually gained both marks, but a common misconception was to think that $10e^{-\frac{1}{8}}$ should be added to the answer to part (a).

Part (c) proved a challenging final question, with usually only the very good candidates scoring all three marks. From those who tried to solve this in one stage it was more common to see D = 1 or 10 or 20 or 13.549, instead of than 15.3526.., substituted into $x = De^{-\frac{1}{8}}$. Many candidates split up the doses but this, unfortunately, often led to a complex expression in T, $3 = 10e^{-\frac{T}{8}} + 10e^{-}$, which only the very best candidates were able to solve. One mark was a common score for this part.

5. Calculator work was generally accurate in this question and it was encouraging to see most candidates give their answers to the required degree of accuracy. The vast majority of candidates gave the correct answer of 425°C in part (a). Many candidates were able to substitute $T = 300$ in part (b) and correctly change an equation of the form $e^a = b$ to $a = \ln b$ Weaker candidates showed a lack of understanding of logarithms by failing to simplify their initial equation to the form $e^a = b$ and using an incorrect statement of the form $a = b + c \Rightarrow \ln a = \ln b + \ln c$ Not all candidates understood the need to differentiate in part (c) and found the gradient of a chord instead of finding $\frac{dT}{dt}$. The most common error made by candidates who did differentiate was to give the differential as $-20\ te^{-05.0t}$ Candidates often had difficulty giving precise explanations in part (d). Although many referred to the +25 term in their answers, far fewer gave adequate reasons as to why this meant that the temperature could never fall to 20° C, particularly with regard to $e^{-05.0\ t} > 0$. Lack of understanding of the concept of limit led some to write (in words or symbols) $T \geq 25$ rather than $T > 25$.

6. (a) The value of a was calculated accurately by most candidates, but a significant group did not substitute t = 0. Also a number of answers were produced where a = 0.12 was substituted to give p=300, and this was not given full credit.

(b) Candidates could not always cope with the algebra manipulation and some found difficulty using logs correctly; e.g. writing 1850 = 114 $e^{0.2t}$ then ln1850 =ln114 $\ln e^{0.2t}$.

(c) Candidates frequently did not show convincing work here, with the answer following from the question with no working in between. It was necessary to show the division of numerator and denominator by e^0.2t to justify getting the given result.

(d) Candidates rarely used the concept of limiting values. Many candidates simply substituted P = 2800 in the formula given in c) and showed that $e^{-0.2t} = 0$, which was insufficient without further statements. There were number of inequality errors seen There were however some excellent solutions which clearly indicated that $e^{-0.2t} > 0$ implied that the denominator was >0.12 and that the fraction was < 2800. Some illustrated their solution with a graph of an increasing function tending to an asymptote.

7. The majority of candidates gained the marks in part (a) although a few did not giver their answer to 3 significant figures. Part (b) was well answered by those who understood logs. Most did combine the logs correctly, but some did still split it up into $\log_2 2x + \log_2 x$ or $\dfrac{\log_2(2x+1)}{\log_2 x}$. Many candidates found the combination of logs and trig functions beyond them. sin x = -1/sec x was a frequent indicator of poor understanding, though many did display they knew sec x = 1/cos x. Quotient lines often slipped, ln 1/cos x becoming 1/ln cos x.

8. The purpose of this sketch was to show the overall shape and orientation of the graph. Many sketches seen were indistinguishable from a straight line and some candidates had clearly been using graphic calculators without being how taught to use appropriate scale factors. Many who did produce curves failed to note that the domain was restricted to x = 0. Part (b) was well done although it was surprising to see at this level a substantial number who carried out ten separate multiplications by 1.04. This method did gain the two marks but was not good time management. A few calculated expressions like 832^{10} and failed to notice that £1.589×10^{29} was an unreasonable answer. In part (c) many could not handle the fact that no initial sum was specified and were unable to reduce the problem to an equation in a single variable. Those who did obtain $1.04^x = 2$ were usually able to complete the question.

9. No Report available for this question.